Motion

Motion:- The change of the position of an object with respect to a point object is called motion.

Point object :- The object is said to be a point object if its dimensions i.e length, breadth and height are very small as compared to the distance travelled by the object.

Distance (s) :- The difference between two places irrespective of any direction is called distance. It is a scalar quantity with S.I unit m.

Displacement (s) :- The shortest distance between two places in a particular direction is called displacement. It is a vector quantity with S.I unit m.

Speed :- The distance travelled by a moving object in unit time is called speed. It is a scalar quantity.

$$\text{speed} = \frac{\text{Distance}}{\text{time}}$$ with S.I unit m/sec.

Average speed :- The total distance travelled by an object with the total time taken to covere the distance

Mathematically, Average speed $= \frac{\text{total distance}}{\text{total time}}$

Velocity :- The total distance travelled by an object in a particular direction per unit time is known as velocity.

$$V = \frac{s}{t}$$

where, v = velocity
s = displacement
t = time

Uniform / constant velocity :- An object is said to have uniform velocity, if it travels in a straight line and covers equal distances in equal time interval.

Average velocity :- The ratio of total displacement to the total time taken is known as average velocity.

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}}.$$

$$\Rightarrow \text{average velocity} = \frac{u+V}{2}$$

where, u = initial velocity
V = final velocity.

Acceleration :- The rate of change of velocity with time is known as acceleration, denoted by a

So, Acceleration = $\frac{\text{change in velocity}}{\text{time}}$

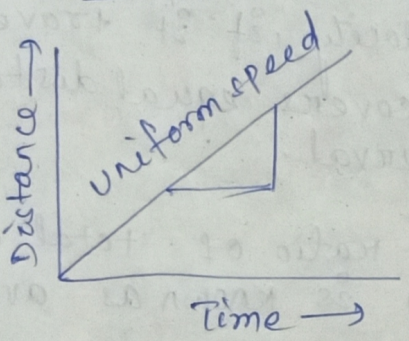
$$\Rightarrow a = \frac{V-u}{t}.$$

(change in velocity = V-u)

Uniform Acceleration :- If the rate of change of velocity remains same throughout the motion.

Non-uniform acceleration :- If the rate of change of velocity is unequal in equal interval of time.

1. Distance-time Graph for uniform speed

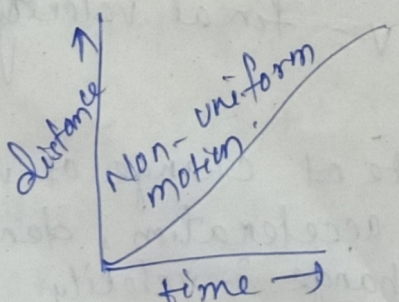


slope of the Graph indicates speed.

$$\text{slope} = \frac{\text{Perpendicular}}{\text{Base}}$$

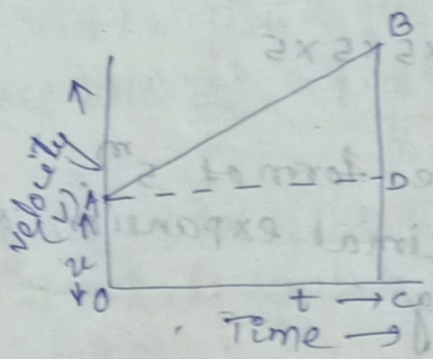
$$\Rightarrow \frac{s}{t} = v.$$

2. Distance-time graph for non-uniform motion



Equation of motion by Graphical Method

(i) consider an object having an initial velocity 'u' at A, then its velocity changes with uniform rate from A to B. so there is a uniform acceleration from A to B.



(i) Equation for velocity - time relation -

$$V = u + at$$

From the graph,

initial velocity, $u = OA$

Final velocity, $V = BC$

$OC = AD = t$.

But from the graph, $BC = BD + CD$.

$\Rightarrow BC = BD + OA \quad (\because CD = OA)$

$\Rightarrow V = BD + u \rightarrow (i)$

Acceleration 'a' = slope of line AB

$\Rightarrow a = \frac{BD}{AD} = \frac{BD}{t}$.

$\Rightarrow BD = at \rightarrow (ii)$

Putting BD in eq (i) we get,

$V = u + at$ - Newton's 1st equation.

(ii) Equation for position - time relation

$$s = ut + \frac{1}{2}at^2$$

Let the object travels a distance 's' in time 't'.

So the distance travelled = area below the graph AB.

$\Rightarrow$ s = area $OABC$

= area of $\square OACD$ + area of $\triangle ABC$

$= OA \times OC + \frac{1}{2} AD \times BD$.

$= u \times t + \frac{1}{2} t \times at$ (As $BD = at$)

$\Rightarrow \boxed{s = ut + \frac{1}{2}at^2}$ - 2nd equation of motion.

(iii) Equation for position - velocity relation

$$v^2 = u^2 + 2as$$

The distance covered by the object is equal to the area of trapezium $OABC$.

$$s = \frac{1}{2}(OA + BC) \times OC$$

$$= \frac{1}{2}(u + v) \times t \rightarrow (1)$$

From Newton's 1st equation $v = u + at$

$\Rightarrow at = v - u$

$\Rightarrow t = \frac{v-u}{a}$

Putting the value of 't' in eq^n (1) we get.

$s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right)$ $\quad \left[a^2 - b^2 = (a+b)(a-b) \right.$

$= \frac{1}{2}\left(\frac{v^2-u^2}{a}\right)$

$\Rightarrow 2as = v^2 - u^2$

$\Rightarrow \boxed{v^2 = u^2 + 2as}$ - 3rd equation of motion.